Appendix C – Assessment of flexibility on the relationship between the control column position and elevator deflection

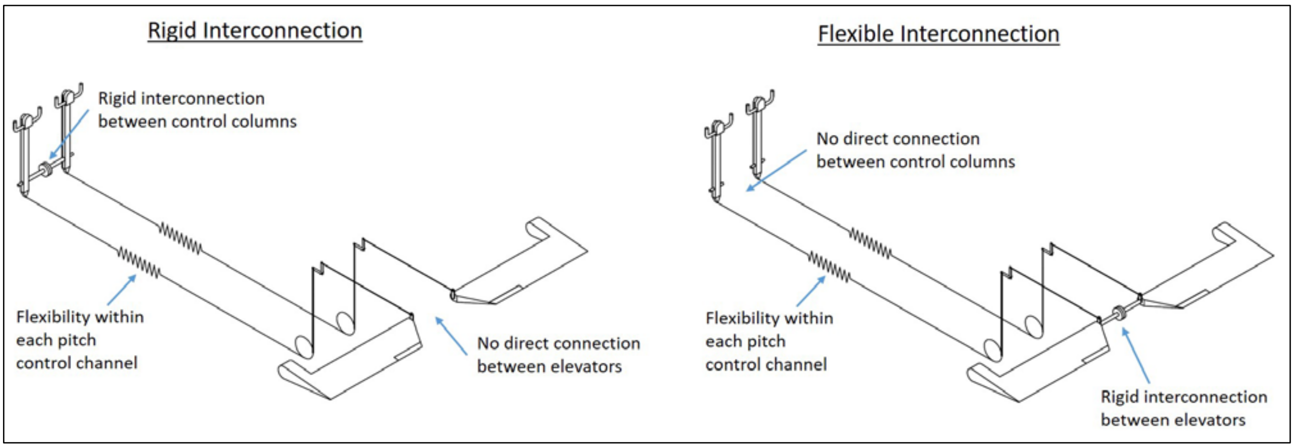
Pitch control system design philosophies

Review of a number of pitch control systems has identified that there are two primary configurations of the pitch control system. Those where the interconnection between the left and right channels is located between, or close to, the:

- control columns with no control cable between the control columns. These systems have an essentially rigid interconnection between the control columns.
- elevators. In these systems, such as in the ATR 72, there were typically long control cable runs located between each control column. This results in a flexible interconnection between the control columns.

For the purposes of this analysis, two models were developed to represent the 'rigid interconnection' and 'flexible interconnection', as shown in Figure C1.

Figure C1: Simplified models of rigid and flexible interconnection systems



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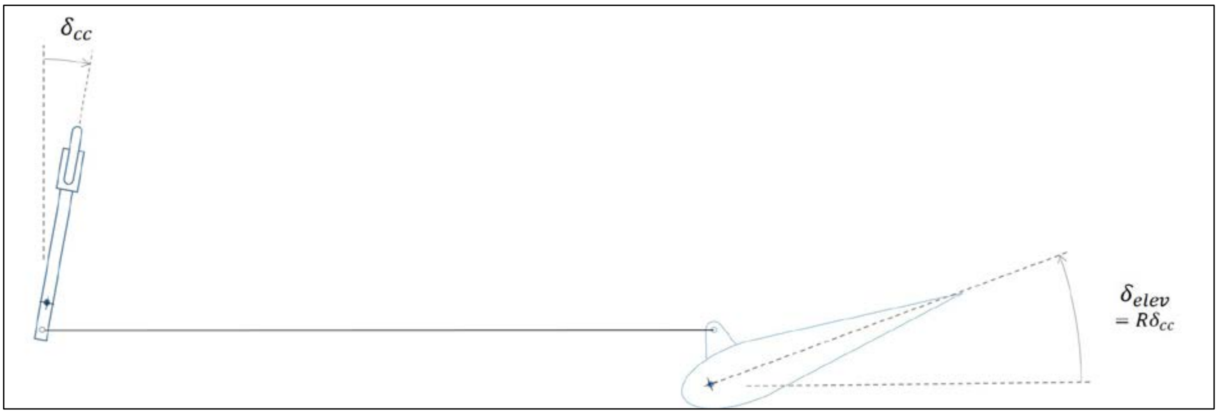
In both cases, the system has a left and a right control station (captain's and first officer's control stations). As can be seen, the only difference between these models is the location of the interconnection. This direct interconnection includes the pitch uncoupling mechanism, which within normal control operating loads can be considered to be effectively rigid.

Analysis of the system behaviours

In any pitch control system, there is typically an amplification of the control column movement at the elevator. This amplification, or control ratio (R), is typically fixed by the geometry of the system and is defined in this analysis as the ratio of the control column deflection (δ_{CC}) to the elevator deflection (δ_{elev}), or:

$$R = \frac{\delta_{elev}}{\delta_{cc}} \quad \text{(Equation 1)}$$

Figure C2: Control ratio



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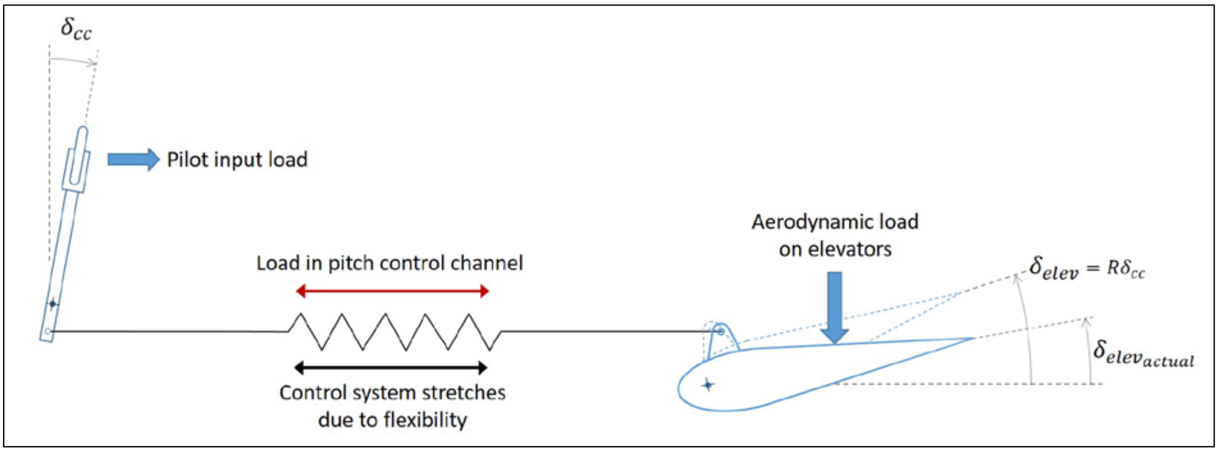
However, in either of the two system models, there is flexibility between the control column and the elevator. When there is no load on the system, such as when the controls are moved on the ground, the system is essentially rigid and the elevators will deflect in accordance with the control ratio. However, when there is a load through the system, the flexibility essentially results in a stretching of the system between the control column and the elevator. This stretching results in the elevator deflecting less than would be expected from the system's control ratio (Figure C3).

Figure C3: Effect of system flexibility on the elevator deflection



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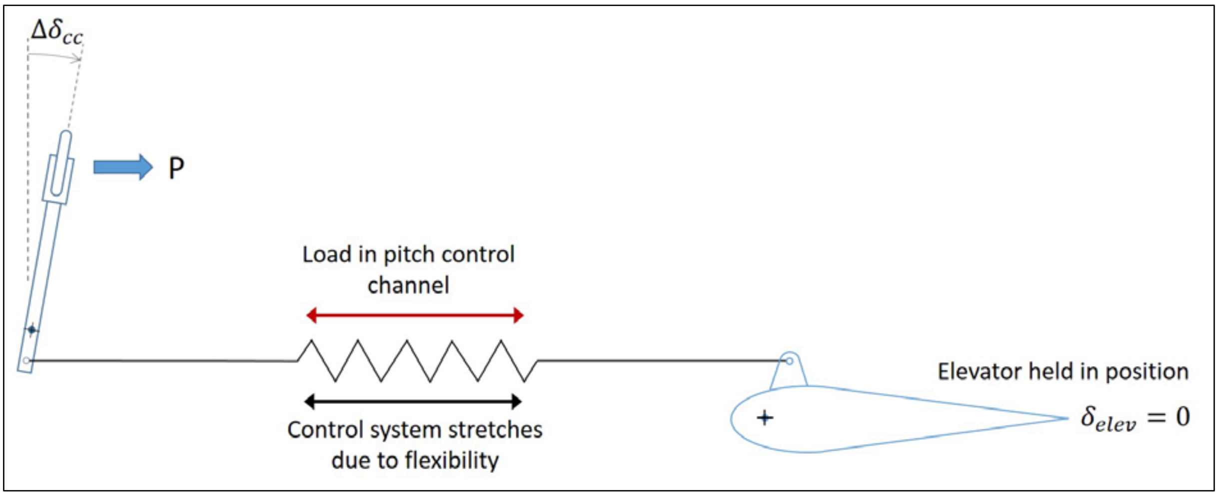
Assuming that the flexibility is linear, the stretch in the system will be proportional to the load in the system. In reality, the system is constructed of multiple elements such as cables, push-pull rods and bellcranks, with different local control ratios. Thus, the internal loads within the system may vary throughout its length. However, the fixed geometry and linearity of the flexibility, the overall flexibility can be represented as stiffness on the control deflections. For this analysis we will represent the overall system stiffness (K) as the amount of control column deflection due to flexibility ($\Delta\delta_{cc}$) per unit of control input force (P).

$$K = \frac{\Delta\delta_{cc}}{P} \qquad \text{(Equation 2)}$$

The deflection in the control column due to the system flexibility only can be demonstrated by examining the case where the elevator is held in the undeflected position and a force applied to the control column (Figure C4).

Figure C4: Control column deflection due to system flexibility



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Therefore, the control column deflection required to achieve a given elevator deflection (δ_{elev}) is the sum of the control column deflection required to deflect it to that position based upon the control ratio[123] (δ_{cc_0}) and the amount of control column deflection due to the system flexibility.

$$\delta_{cc} = \delta_{cc_0} + \Delta\delta_{cc} \qquad \text{(Equation 3)}$$

However, from Equations 1 and 2, we can see that:

$$\delta_{cc} = \frac{\delta_{elev}}{R}, \text{ and}$$

$$\Delta\delta_{cc} = K \times P = KP$$

Hence, the control column deflection can be represented as:

$$\delta_{cc} = \frac{\delta_{elev}}{R} + KP \qquad \text{(Equation 4)}$$

Rigid interconnection

The behaviour of the system with the rigid interconnection can be represented using the model in Figure C5, where,

P_L and P_R are the pilot input loads applied to the left and right control columns

δ_{cc_L} and δ_{cc_R} are the left and right control column deflections

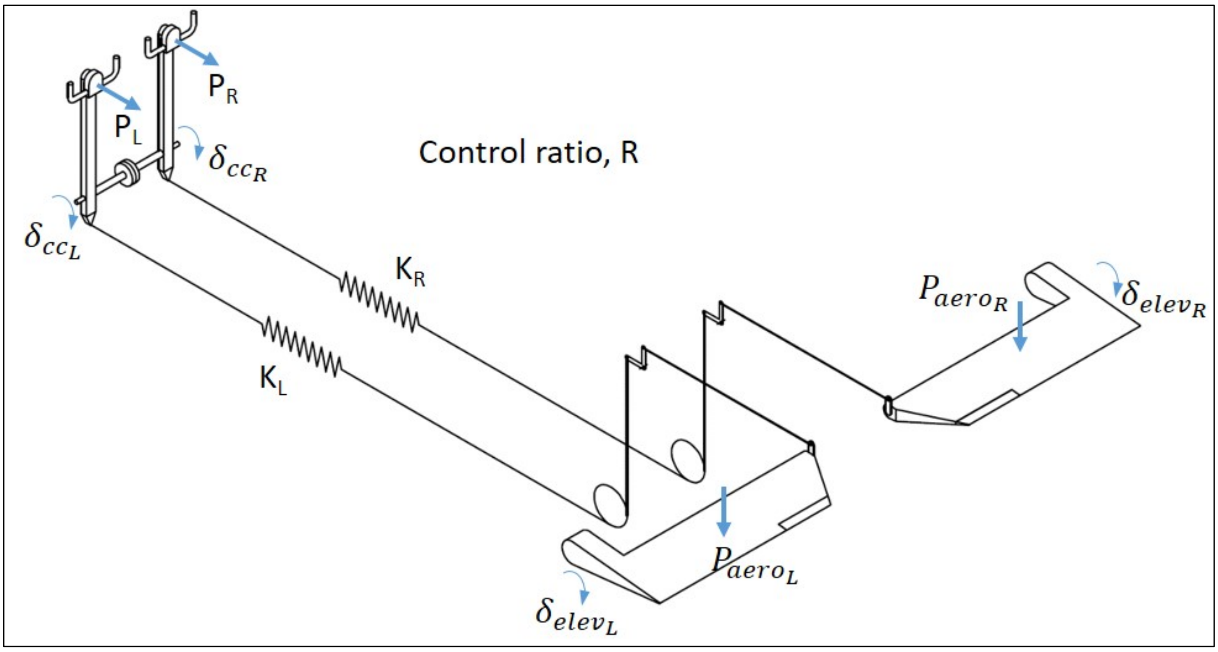
K_L and K_R are the stiffness of the left and right control channels

P_{aero_L} and P_{aero_R} are the aerodynamic loads on the elevators due to their deflection

δ_{elev_L} and δ_{elev_R} are the left and right control column deflections

[123] This is the same as having no load on the system, so represented with a '0' subscript.

Figure C5: Rigid interconnection control system model



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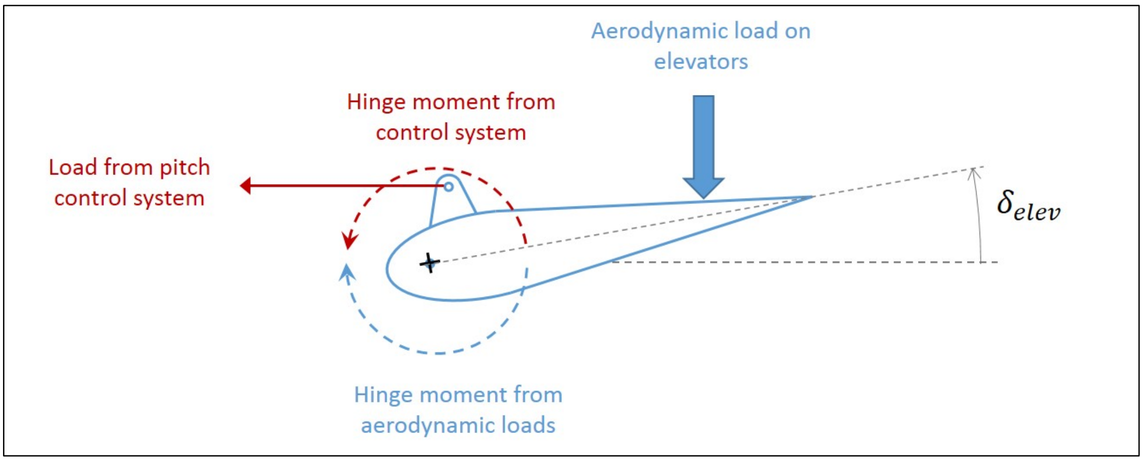
The elevator will move whenever the moments generated by the aerodynamic loads are not balanced by the moment from the control input. The elevator will reach a stable elevator deflection when these moments are balanced (Figure C6**Error! Reference source not found.**). Increasing the elevator deflection increases the aerodynamic load, so the amplitude of the elevator deflection is based upon the magnitude of the control input load.

Figure C6: Balance of moments from aerodynamic and control input loads



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Assuming that for normal elevator deflections, the effective location of the aerodynamic load remains fixed,[124] and given the fixed control system geometries, we can represent the hinge moment balance as a force balance between the control input loads and the aerodynamic loads in the following manner:

$$P_L + P_R = \frac{1}{R}\left(P_{aero_L} + P_{aero_R}\right) \quad \text{(Equation 5)}$$

[124] It is actually a pressure distributed over the surface, but can be represented as an equivalent force acting at a distance from the hinge.

Because there is a rigid connection between the left and right control columns, both control columns will always deflect by the same amount (δ_{cc}),

$$\delta_{cc_L} = \delta_{cc_R} = \delta_{\mathrm{cc}}$$

Because both control columns deflect by the same amount, the load through each pitch channel will be distributed proportional to the relative stiffness of the pitch channel to the overall stiffness. For example in the left channel,

$$Load\ through\ left\ channel = (P_L + P_R) \times \frac{K_L}{K_L + K_R} \quad \text{(Equation 6)}$$

Assuming that the stiffness of both sides is equal,

$$K_{\mathrm{L}} = K_R = K$$

So from Equation 6, the load through each pitch channel is half the total load applied to the control columns. Also, assuming that the elevators and the airflow over them are perfectly symmetrical between the left and right sides, the aerodynamic loads will be the same and they will deflect by the same amount.

$$P_{aero_L} = P_{aero_R} = P_{aero}$$

$$\delta_{elev_L} = \delta_{elev_R} = \delta_{\mathrm{elev}}$$

Hence, when everything about the system is symmetrical, the force balance becomes:

$$P_L + P_R = \frac{1}{R}(P_{aero} + P_{aero}) = \frac{2}{R}P_{aero} \quad \text{(Equation 7)}$$

The elevator hinge moment (H_e), being generated by aerodynamic pressures, is effected by the air density (ρ), airspeed (V) and elevator area (S_e) and elevator chord (c_e). The hinge moment is often represented as a coefficient (C_H), defined as,

$$C_H = \frac{H_e}{\frac{1}{2}\rho V^2 S_e c_e}$$

Rearranging this to determine the hinge moment,

$$H_e = \frac{1}{2}\rho V^2 S_e c_e \times C_H \quad \text{(Equation 8)}$$

For comparative purposes, we can assume that the air density and airspeed remain constant. S_e and c_e are fixed geometries, so the elevator hinge moment coefficient is proportional to the hinge moment only.

The hinge moment coefficient is a function of the tailplane incidence, trim tab deflection and elevator deflection. For this comparative analysis, we will assume that the tailplane incidence and trim tab effects are the same, so we are only concerned with the elevator deflection effects. Because we are also assuming that the only differences between the rigid and flexible interconnected systems is the location of the interconnection, the variation in elevator hinge moment with elevator deflection is the same for both systems. Assuming a linear variation of hinge moment coefficient to elevator deflection, the hinge moment, and hence the aerodynamic load, will be proportional to the elevator deflection. Hence the aerodynamic load on the elevator can be represented as,

$$P_{aero} = C_{\delta_e} \times \delta_{elev} \quad \text{(Equation 9)}$$

Where, C_{δ_e} is the proportionality constant in aerodynamic force per degree of elevator deflection.

Combining Equations 7 and 9,

$$P_L + P_R = \frac{2}{R}P_{aero} = \frac{2}{R}C_{\delta_e} \times \delta_{elev} \quad \text{(Equation 10)}$$

For dual control inputs, the control force (P) through one control channel was shown to be half of the combined load ($P_L + P_R$). Substituting this into Equation 4,

$$\delta_{cc} = \frac{\delta_{elev}}{R} + KP$$

$$\delta_{cc} = \frac{\delta_{elev}}{R} + K\frac{1}{2}(P_L + P_R)$$

Substituting in Equation 10,

$$\delta_{cc} = \frac{\delta_{elev}}{R} + \frac{\mathrm{K}}{2}\frac{2}{R}(C_{\delta_e} \times \delta_{elev})$$

$$\delta_{cc} = \left[\frac{1 + KC_{\delta_e}}{R}\right] \delta_{elev}$$

Rearranging to put in terms of δ_{elev},

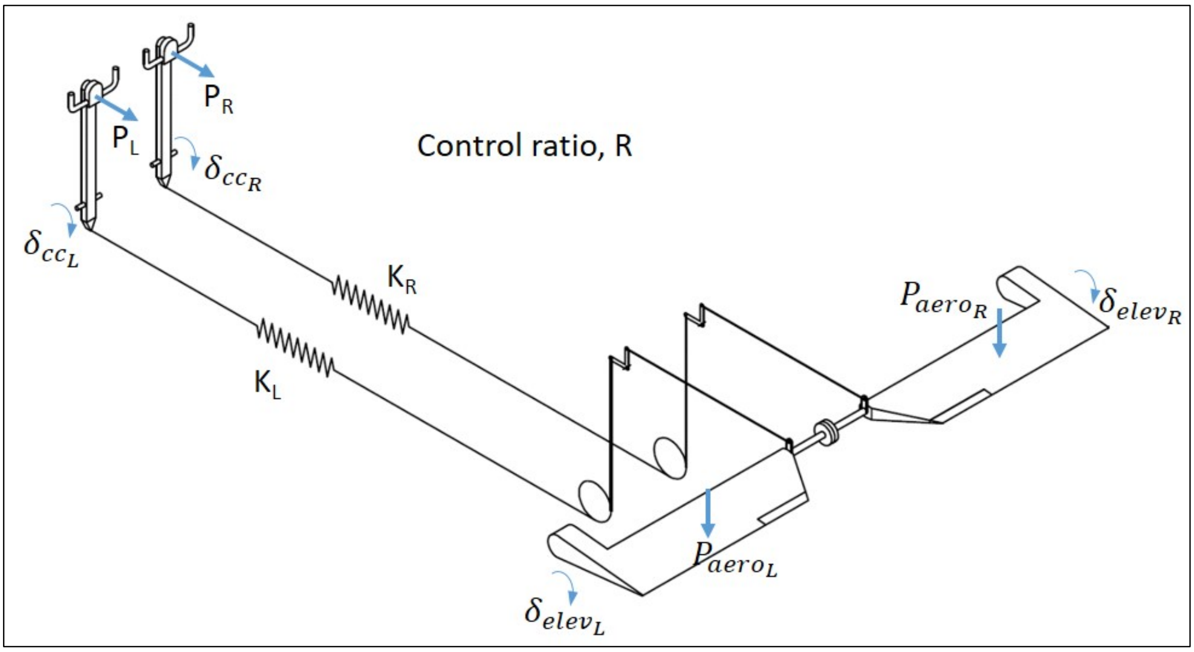
$$\delta_{elev} = \left[\frac{R}{1+KC_{\delta_e}}\right] \delta_{cc} \qquad \text{(Equation 11)}$$

The factors R, K and C_{δ_e} are all constants, so the elevator deflection is a function of the control column deflection only. As such, there is a direct correlation between the control column position and the elevator position. Also, because of the rigid interconnection, both control columns move by the same amount.

Flexible interconnection

The behaviour of the system with a flexible interconnection can be represented by the model in Figure C7.

Figure C7: Flexible control system model



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Because the elevators are rigidly connected, both elevators will have the same deflection,

$$\delta_{elev_L} = \delta_{elev_R} = \delta_{\mathrm{elev}}$$

Similar to the rigid interconnection, the amplitude of the elevator deflection is based upon the balance of the aerodynamic and pilot input loads. Assuming perfect symmetry between the left and right elevators, Equation 7 applies.

$$P_L + P_R = \frac{2}{R} P_{aero}$$

The same aerodynamic principles apply, so the relationship between elevator deflection and the resulting aerodynamic load applies. Equation 10 can then be rearranged to present the load balance from the perspective of each control column.

$$P_L = \frac{2}{R} C_{\delta_e} \times \delta_{elev} - P_R \quad \text{(Equation 12a)}$$

$$P_R = \frac{2}{R} C_{\delta_e} \times \delta_{elev} - P_L \quad \text{(Equation 12b)}$$

Because the control columns are not rigidly connected, δ_{cc_L} is not necessarily the same as δ_{cc_R}, and the control system stretch in each channel may be different. So the control system deflections due to system stretch are, assuming that the left and right systems have the same stiffness (K_L = K_R = K):

$$\Delta\delta_{cc_L} = KP_L \quad \text{(Equation 13a)}$$

$$\Delta\delta_{cc_R} = KP_R \quad \text{(Equation 13a)}$$

Combining with Equations 4, 12a and 12b, we see that the left control column deflection can be presented as:

$$\delta_{cc_L} = \frac{\delta_{elev}}{R} + K \times \left[\frac{2}{R} C_{\delta_e} \times \delta_{elev} - P_R\right]$$

$$\delta_{cc_L} = \frac{\delta_{elev}}{R} + K \times \frac{2}{R} C_{\delta_e} \times \delta_{elev} - K \times P_R$$

Rearranging to put in terms of the elevator deflection,

$$\delta_{elev} = \frac{R}{1+2KC_{\delta_e}} \left[\delta_{cc_L} + KP_R\right] \quad \text{(Equation 14a)}$$

Similarly, for the right channel,

$$\delta_{elev} = \frac{R}{1+2KC_{\delta_e}} \left[\delta_{cc_R} + KP_L\right] \quad \text{(Equation 14b)}$$

Again, the factors R, K and C_{δ_e} are all constants; however, in this case, the elevator deflection is a function of both the control column deflection and the opposite control column load.

The implications of this are that the elevator position can be changed by either control column without a respective change in the other control column. For example, the left control column could be held in position, and an input be made on the right. Similar to the rigid interconnection, the left pilot will feel a change in the force on the control column, but with a flexible interconnection, δ_{cc_L} can remain the same, but the elevator position will change due to the force in the right channel.

The more flexible the system, that is the larger value of K, the greater the effect the other pilot's inputs will have on the elevator position.

This means that for an aircraft with a flexible interconnection, the pilot flying will not get the same consistent feedback on the state of the elevator from the control column movement.

Note also, for the case of single pilot inputs, equation 14 reduces to a form similar to the rigid control case, except that there is an additional factor of 2 in the denominator of the constant factor. This is because the load is all through one channel, rather than shared between the left and right channels, as is the case for the rigid interconnection. This means that the effects of flexibility are more pronounced in a control system with a flexible interconnection between the left and right pitch channels.

System gain

The aerodynamic load on the entire tailplane is a function of the geometry of the tailplane, the angle of attack, environmental conditions (density and airspeed), and the elevator deflection. The tailplane geometry is fixed, and in the short term, environmental conditions are constant, so only the angle of attack and elevator deflection will change. Also, when the elevator is initially deflected, the angle of attack will change only due to the elevator deflection.[125] As such, when examining the short term, the aircraft response can be represented by the elevator deflection. The system gain can then be expressed in terms of the ratio of the elevator deflection to the control column deflection, due to its relationship to the elevator position.

$$System\ gain = \frac{\delta_{elev}}{\delta_{cc}} \quad \text{(Equation 15)}$$

From Equation 11, the system gain for a control system with a rigid interconnection is:

$$System\ gain\ (rigid) = \frac{\delta_{elev}}{\delta_{cc}} = \left[\frac{R}{1+KC_{\delta_e}}\right] \quad \text{(Equation 16)}$$

From Equations 14a and 14b, the system gain from the viewpoint of each pilot is:

$$System\ gain\ (flex, left) = \frac{\delta_{elev}}{\delta_{cc_L}} = \left[\frac{R}{1+2KC_{\delta_e}}\right]\left[1 + \frac{KP_R}{\delta_{cc_L}}\right] \quad \text{(Equation 17a)}$$

$$System\ gain\ (flex, right) = \frac{\delta_{elev}}{\delta_{cc_R}} = \left[\frac{R}{1+2KC_{\delta_e}}\right]\left[1 + \frac{KP_L}{\delta_{cc_R}}\right] \quad \text{(Equation 17b)}$$

Thus, for a system with a rigid interconnection, the system gain is effectively constant. However, for a system with a flexible interconnection, the system gain may be different for each control station and is a function of both the control column deflection and the control input force on the opposing control column.

Conclusions

The location of the interconnection between the left and right pitch control channels effects the rigidity of the interconnection between the left and right control columns. When the interconnection is located at or close to the elevators, there is flexibility between the control columns. Flexibility between the control columns changes the relationship between the control column position and the elevator position in a complex manner.

When there is a rigid interconnection between the control columns, both control columns will move in unison and there is a constant relationship between the control column and elevator deflections. This provides a consistent feedback between the control column position and the elevator deflection. That is, the elevator will not move without a corresponding change in the control column position. However, when there is flexibility between the control columns, the elevator position is a function of the control column position and the force on the other control column. For example, if one control column is held, and a force applied to the other control column, the elevator will also move, and the relationship between the control column position and the elevator position is lost.

In terms of the system gain, the rigid interconnection results in a gain that is consistent between

[125] The angle of attack of an aerofoil is referenced to its chord line (an imaginary line between the leading and trailing edges of an aerofoil). Deflection of the elevator changes the chord line, and as such changes the angle of attack of the aerofoil.

single and dual control inputs and between the control stations. However, the system gain of a flexible interconnect results in a gain that differs between single and dual inputs and, unless the control inputs are identical, between the control stations.